

## Problems in Magnetic Properties of Materials

Notations used:

$H$ : Magnetic field strength

$B$ : Magnetic flux density

$I$ : Intensity of Magnetization (Please note that, in text book, notation,  $M$ , is used for Intensity of magnetization)

$i$ : Current

$\mu_0$  : Permeability of free space

$\mu$  : Permeability of medium other than free space.

$\mu_r$  : Relative permeability

$\mu_s$  : Saturation magnetization

$\chi$  : Susceptibility

$N_A$ : Avogadro number

Any other symbol used is written wherever required.

1. A coil of wire 0.25 m long and having 400 turns carries a current of 15 A. Find magnitude of magnetic field strength. Compute the flux density,  $B$ , if the coil is in vacuum.

Sol: Magnetic field strength,  $H = \frac{Ni}{l}$  where  $N$  is number of turns,  $i$  current passing in the coil and  $l$  is length of the coil.

$$H = \frac{400 \times 15}{0.25} = 24000 \text{ A/m}$$

Flux density in vacuum,  $B_0 = \mu_0 H = 4\pi \times 10^{-7} \times 24000 = 0.030144 \text{ T}$

2. Compute the flux density inside a bar of chromium that is positioned within a coil of wire 0.25 m long and having 400 turns, carrying a current of 15 A. Magnetic susceptibility of chromium is  $3.13 \times 10^{-4}$ . Also compute magnetization of bar of chromium.

Sol.: Magnetic field strength,  $H = \frac{Ni}{l} = \frac{400 \times 15}{0.25} = 24000 \text{ A/m}$

Relative permeability,  $\mu_r = \chi + 1 = 3.13 \times 10^{-4} + 1 = 1.000313$

and permeability,  $\mu = \mu_r \mu_0 = 1.000313 \times 4\pi \times 10^{-7} = 12.56 \times 10^{-7} \text{ H/m}$

Flux density inside the bar of chromium,

$$B = \mu H = 12.56 \times 10^{-7} \times 24000 = 301440 \times 10^{-7} \text{ T}$$

Magnetization,  $I = \frac{\chi}{H} = \frac{3.13 \times 10^{-4}}{24000} = 0.13 \times 10^{-7} \text{ A/m}$

3. The magnetic flux density within a bar of some material is 0.63 Tesla at an H field of  $5 \times 10^5$  A/m. Compute the following for this material: (a) Magnetic permeability, (b) Magnetic susceptibility, (c) Type of magnetism that you suggest being displayed by the material with reasons.

Sol.: (a) Magnetic permeability,  $\mu = \frac{B}{H} = \frac{0.63}{5 \times 10^5} = 0.126 \times 10^{-5} \text{ H/m}$

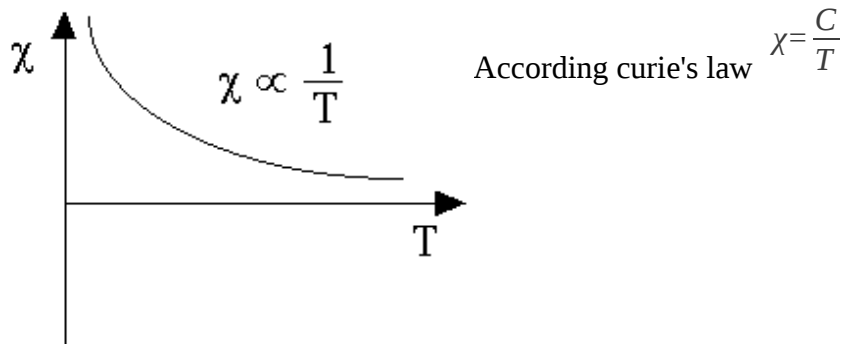
(b) Magnetic susceptibility,  $\chi = \mu_r - 1 = \frac{\mu}{\mu_0} - 1 = \frac{0.126 \times 10^{-5}}{4\pi \times 10^{-7}} - 1 = 1.003185 - 1 = 0.003185$

(c) Type of magnetism: Paramagnetism since the magnetic susceptibility is positive and low in magnitude.

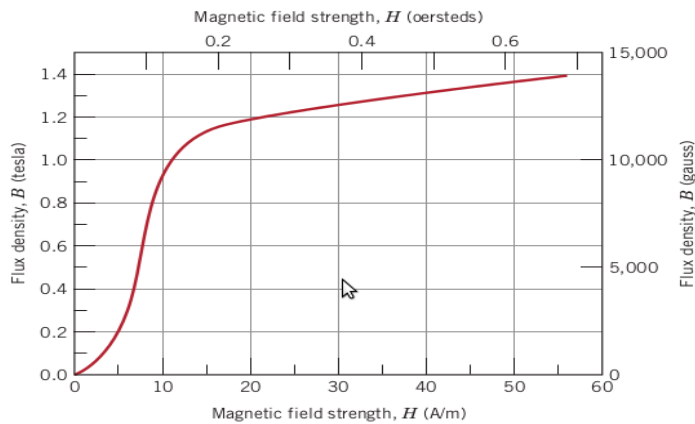
4. Magnetic susceptibility of Chromium and Aluminium are  $3.13 \times 10^{-4}$  and  $2.07 \times 10^{-5}$  respectively at room temperature. Identify the material that can easily be magnetized. Draw and explain the plot showing temperature dependence of susceptibility of paramagnetic material

Sol.: Material that has larger susceptibility can be magnetized easily. So as per the data, Chromium can be magnetized easily compared to Aluminium.

Temperature dependence of susceptibility of paramagnetic materials is shown below.



5. A coil of wire 0.5 m long having 20 turns carries a current of 1.0 A. (a) Compute flux density if the coil is within the vacuum. (b) If a bar of iron-silicon alloy that shows B-H relation as shown in below figure, is positioned in the coil, Find flux density within the coil.



Sol.: Magnetic field strength,  $H = \frac{Ni}{l} = \frac{20 \times 1.0}{0.5} = 40 \text{ A/m}$

Magnetic flux density in vacuum,  $B_0 = \mu_0 H = 4\pi \times 10^{-7} \times 40 = 502.4 \times 10^{-7} \text{ T}$

Magnetic flux density within the coil when a bar of iron-silicon alloy is placed in the coil:  
From the graph, the magnetic flux density at  $H = 40 \text{ A/m}$  is 1.3 Tesla. So magnetic flux density is 1.3 T.

6. A ferro magnetic material has remanance of 1.0 Tesla and a coeefcivity of 15,000 A/m. Saturation is achieved at a magnetic field strength of 25,000 A/m at which flux density is 1.25 Tesla. Using this data sketch entire Hysteresis curve in the range,  $H = -25,000$  to  $+25,000 \text{ A/m}$ . Scale and lable both the coordinate axes.

Note: Left to student's home work

7. The data shown below are for plain carbon steel alloy. Using this data, (i) draw a graph between B and H. (ii) Find the values of initial permeability and relative permeability.

$H$ (A/m)	$B$ (teslas)	$H$ (A/m)	$B$ (teslas)
0	0	80	0.90
15	0.007	100	1.14
30	0.033	150	1.34
50	0.10	200	1.41
60	0.30	300	1.48
70	0.63		

Note:

Left to student's home work

8. An iron bar magnet having coercivity of 7000 A/m is to be demagnetized. If the bar is inserted within a cylindrical wire coil 0.25 m long and having 150 turns, find electric current required to generate necessary magnetic field.

Sol. Required electric current,  $i = \frac{HI}{N} = \frac{7000 \times 0.25}{150} = 11.66 \text{ A/m}$

9. Compute the saturation magnetization and saturation flux density for iron (atomic weight: 55.8) which has a net magnetic moment per atom of 2.2 Bohr magnetons and a density of 7.87 g/cm<sup>3</sup>

Sol.

Let N is number of atoms per unit volume of the magnet.

$$N = \frac{\rho N_A}{A} = \frac{7.87 \times 6.023 \times 10^{23}}{55.8} = 0.849 \times 10^{23} \text{ atom/cm}^3$$

Here  $\rho$  is density,  $N_A$  is Avogadro Number and A is atomic weight.

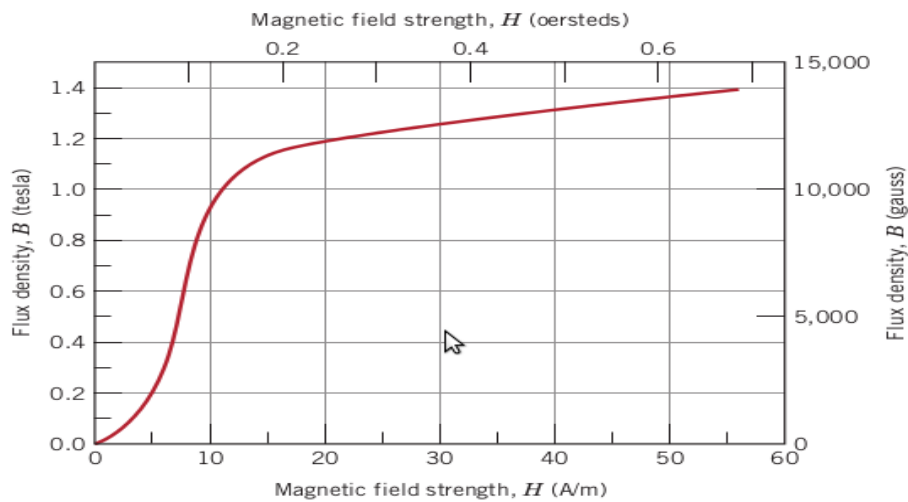
Magnetic moment per atom, i. e.

$$\mu_{atom} = 2.2 \mu_B = 2.2 \times 9.27 \times 10^{-24} \text{ A-m}^2 = 20.394 \times 10^{-24} \text{ A-m}^2 \quad (\text{as per the give data})$$

Saturation magnetization,

$$\mu_s = N \mu_{atom} = 0.849 \times 10^{23} \times 20.394 \times 10^{-24} = 17.314 \times 10^{-1} = 1.7314 \text{ A-m}^2$$

10. A bar of an iron-silicon alloy having the B-H behavior shown below is inserted within a coil of wire 0.4 m long and having 50 turns, through which passes a current of 0.1 A. (a) Find magnetic flux density, B, within the bar. (b) At this field, find permeability, relative permeability, Susceptibility and magnetization.



Sol.: Magnetic

field strength,  $H = \frac{Ni}{l}$  where  $N$  is number turns,  $i$ : current in the coil and  $l$  is length of the coil.

$$H = \frac{50 \times 0.1}{0.4} = 12.5 \text{ amp/m}$$

(a) From the graph, the value of  $B$  corresponding to  $H = 12.5$  amp/m is 1.1 Tesla.

(b) Permeability,  $\mu = \frac{B}{H} = \frac{1.1}{12.5} = 0.088$

Relative permeability,  $\mu_r = \frac{\mu}{\mu_0} = \frac{0.088}{4\pi \times 10^{-7}} = 0.0070064 \times 10^7 = 70064$

Susceptibility,  $\chi = \mu_r - 1 = 70064 - 1 = 70063$

Magnetization,  $I = \chi H = 70063 \times 12.5 = 875787.5 \text{ amp/m}$

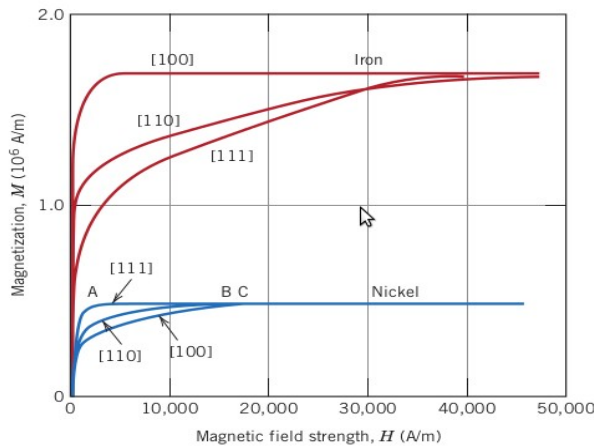
11. A magnetic material has a magnetization of 3300 A/m and flux density of 0.0044 Wb/m<sup>2</sup>. Calculate magnetizing field strength and relative permeability. Identify the type magnetism associated to it.

Sol.: From  $B = \mu_0(H + I)$ , magnetizing field strength,  
 $H = \frac{B}{\mu_0} - I = \frac{0.0044}{4\pi \times 10^{-7}} - 3300 = 3503.185 - 3300 = 203.185 \text{ A/m}$

Relative permeability, 
$$\mu_r = \frac{\mu}{\mu_0} = \frac{B}{H\mu_0} = \frac{0.0044}{203.185 \times 4\pi \times 10^{-7}} = 17.24$$

Type of magnetism : Ferromagnetism

12. Define easy and hard directions of magnetization of ferromagnetic material. In the plot shown above identify easy and hard directions of magnetization in single crystal of Iron and Nickel.



Sol: Left to student's home work.

13. The magnetic field intensity in a piece of a magnetic material is  $10^6$  A/m. If the susceptibility of the material at room temperature is  $1.5 \times 10^{-3}$ , compute flux density and magnetization of material. Also, identify the type magnetism in the material.

Sol.: Flux density:

$$B = \mu_0 (H + I) = \mu_0 H \left( 1 + \frac{I}{H} \right) = \mu_0 H (1 + \chi) = 4\pi \times 10^{-7} \times 10^6 (1 + 1.5 \times 10^{-3}) = 1.257 \text{ T}$$

Magnetization:  $I = \chi H = 1.5 \times 10^{-3} \times 10^6 = 1500 \text{ A/m}$

Type of magnetism: Ferro.

14. Magnetic susceptibility of a material is  $-0.5 \times 10^{-5}$ . If the material is subjected to magnetic field of intensity,  $10^4$  A/m, find magnetic moment per unit volume of the material and magnetic flux density of the material. Also, identify the type of magnetism associated to the material.

Sol.: Magnetic moment per unit volume (Intensity of magnetization) is given by,  

$$I = \chi H = -0.5 \times 10^{-5} \times 10^4 = -0.05 \text{ A/m}^2$$

Magnetic flux density,  $B = \mu_0(H+I) = 4\pi \times 10^{-7}(10^4 - 0.05) = 12.56 \times 10^{-3} = 0.01256 T$

Type of magnetism: Dia.

15. A paramagnetic material has body-centered cubic structure with unit cell edge of 0.25 nm. If the saturation value of magnetization is  $1.8 \times 10^6 A/m$ , calculate the average magnetization contributed per atom in Bohrmagneton.

Sol: Magnetization is magnetic moment per unit volume.

Saturation magnetization =  $1.8 \times 10^6 A/m$  (magnetic moment per unit volume)

magnetic moment per unit cell of side  $a$ ,  $I = 1.8 \times 10^6 \times a^3$

$$I = 1.8 \times 10^6 \times (0.25 \times 10^{-9})^3 = 0.028125 \times 10^{-21} Am^2 = 28.125 \times 10^{-24} Am^2 = 3 \text{ Bohrmagneton}$$

Note: 1 Bohrmagneton =  $9.4 \times 10^{-24} Am^2$

Number of atom in BCC unit cell of side  $a = 8$

Average magnetization per atom =  $3/8$  Bohrmagneton =  $0.375$  Bohrmagneton.

16. The rare earth element gadolinium is ferromagnetic below  $16^\circ C$  with 7.1 Bohrmagneton per atom. Calculate the magnetic moment per gram. Find the value of saturation magnetization. Atomic weight of Gadolinium is 157.26 and its density is  $7.8 \times 10^3 kg/m^3$ .

Sol: 157.26 gm of Gadolinium has Avogadro number of atoms, i. e.  $6.023 \times 10^{23}$  atoms. .

157.26 gm of Gadolinium has magnetic moment of  $7.1 \times 6.023 \times 10^{23}$  Bohrmagneton.

Magnetic moment per gram of Gadolinium =

$$\frac{7.1 \times 6.023 \times 10^{23}}{157.26} = 0.2719 \times 10^{23} \text{ Bohrmagneton}$$

Let  $N$  is number of atoms per unit volume of the magnet.

$$N = \frac{\rho N_A}{A} = \frac{7.8 \times 6.023 \times 10^{23}}{157.26} = 0.298 \times 10^{23} \text{ atom/cm}^3$$

Here  $\rho$  is density,  $N_A$  is Avogadro Number and  $A$  is atomic weight.

Magnetic moment per atom, i. e.

$\mu_{atom} = 7.1 \mu_B = 7.1 \times 9.27 \times 10^{-24} A-m^2 = 65.817 \times 10^{-24} A-m^2 = 65.817 \times 10^{-20} A-cm^2$  (as per the give data)

Saturation magnetization,

$$\mu_s = N\mu_{atom} = 0.298 \times 10^{23} \times 65.817 \times 10^{-20} = 19.61 \times 10^3 = 1961 \text{ A/cm}$$

17. The **B<sub>r</sub>** values of Carbon steel, Tungsten steel, Cobalt steel are 0.95, 1.05 and 0.95 Wb/m<sup>2</sup> respectively. The **H<sub>c</sub>** values of Carbon steel, Tungsten steel and Cobalt steel are 4000, 5200 and 20000 A/m. Using this data Identify the magnetic material which can not be demagnetized easily and which can be easily demagnetized.

Hint: Find the the product **B<sub>r</sub>H<sub>c</sub>** for all the materials. Materials with low magnitudes of the product can be demagnetized easily and with high magnitudes can not be demagnetized easily.

18. The unit cell side of Fe<sub>3</sub>O<sub>4</sub> is about 0.8 nm and there are 8 Fe<sup>++</sup> atoms per unitcell. The Fe<sup>++</sup> has 6 unpaired electrons out of which 5 spins have spins one direction and one electron has spin in opposite direction. Find net magnetic moment of each Fe<sup>++</sup> in terms of Bohr Magneton. Also calculate magnetization of Fe<sub>3</sub>O<sub>4</sub>.

Sol.: Net magnetic moment of each Fe<sup>++</sup> atom, i. e.  $\mu_{atom}$ , is 4 Bohrmagneton.

Magnetization of Fe<sub>3</sub>O<sub>4</sub>,  $I = \frac{n\mu_{atom}}{a^3}$  where n is number of atoms per unit cell, a side of the unit cell.

$$\text{Magnetization, } I = \frac{8 \times 4 \times 9.27 \times 10^{-24}}{(0.8 \times 10^{-9})^3} = 579.375 \times 10^3 \text{ A/m}$$

19. The magnetic field strength in Silicon is 1000 A/m. If the magnetic field susceptibility is  $-0.3 \times 10^{-5}$ , calculate the magnetization and flux density in Silicon.

Sol.: Magnetization,  $I = \chi H = -0.3 \times 10^{-5} \times 1000 = -0.003 \text{ A/m}$

$$\text{Magnetic flux density, } B = \mu_0 (H + I) = 4\pi \times 10^{-7} (1000 - 0.003) = 12.56 \times 10^{-4} \text{ T}$$

20. A paramagnetic salt has  $10^{28}$  ions/m<sup>3</sup> with a magnetic moment of 1 Bohrmagneton per atom. Calculate paramagnetic susceptibility and the magnetization produced in a uniform magnetic field of  $10^6$  A/m when the temperature is 27 °C

Sol. Magnetization,  $I = \text{Bohr magneton per atom} \times \text{number of atoms per unit volume} = 9.27 \times 10^{-24} \times 10^{28} = 9.27 \times 10^4 \text{ A/m}$

$$\text{Magnetic susceptibility, } \chi = \frac{I}{H} = \frac{9.27 \times 10^4}{10^6} = 0.0927$$